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Spin-diffusion equations for semiconductor heterostructures with long-range impurity scattering

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Abstract

We discuss the structure of particle and spin-diffusion equations for semiconductor heterostructures with linear Rashba spin–orbit interaction and long-range impurity scattering. We focus on systems in which the correlation length of the impurity potential is large compared to the wavelength of the particles but small compared to the mean free path. We show that the transport coefficients for such systems are affected differently by the anisotropy of scattering events. Whereas the relaxation times are replaced by momentum relaxation times in transport coefficients coupling spin and spin or charge and charge they are replaced by products of the momentum relaxation time and the scattering time in spin–charge coupling coefficients. This fact leads to a reduction of the magnitude of the spin-accumulation in high-mobility heterostructures in the presence of electric fields.

1. Introduction

Investigations of the impact of the spin–orbit interaction on the spin and charge transport properties of non-magnetic semiconductor heterostructures are of much current interest. In particular the Rashba interaction has received much attention. The Rashba interaction is the conventional spin–orbit interaction with a constant perpendicular electric field in a two-dimensional electron gas. There have been discussions in the literature of a number of interesting effects associated with this interaction which permit complete electric spin-manipulation, like the spin-accumulation in an external electric field (see, e.g., [1–6]), spin-galvanic currents [7, 8], electric field-induced precessions [9–14] or the spin-Hall effect (see, e.g., [15–19]). Many of these effects have also been observed in experiments.

The theoretical description of spin transport processes in semiconductor heterostructures with the Rashba interaction requires the derivation of diffusion equations. Such equations have been derived in a number of papers (see, e.g., [9, 11, 20–22]). The scattering of particles at impurities has been considered as isotropic in all of them. However, this condition is not

satisfied in most semiconductor heterostructures. The scattering is often dominated by ionized impurities in such systems, in particular in heterostructures with remote doping layers. This raises the question of how anisotropic scattering events manifest themselves in spin-diffusion equations.

Charge transport phenomena are characterized by two relaxation times in the presence of anisotropic scattering events, by the scattering time and the momentum relaxation time. The scattering time characterizes the relaxation of the distribution function. It contains all scattering events. The momentum relaxation time takes into account only those events which also relax momentum. Forward scattering events are not contained in the momentum relaxation time. Therefore, the momentum relaxation time is longer than the scattering time.

Spin-charge coupling effects often result from the interplay between both mechanisms. The spin-accumulation in an external field, for example, is determined by the interplay between spin generation and decay of magnetization. Spin generation results from the tendency to restore the properties of the spectrum after scattering events. The characteristic property restored is that the spin of the particles is transverse to their momentum. All scattering events give rise to deviations from this property, not only those which relax momentum. Therefore, the system can be well in equilibrium (or in a stationary state) with respect to spin without being in equilibrium (or in a stationary state) with respect to momentum. This fact suggests that spin generation is related to the scattering time. The decay of the magnetization, on the other hand, is determined by the Dyakonov-Perel mechanism [23], and thus is coupled to momentum relaxation. Thus, it is conceivable that both relaxation times appear in spin-charge coupling terms.

It is the purpose of the present paper to check whether signatures of the scattering time are also reflected in spin-charge coupling terms or not. To this end we consider a system with a linear Rashba interaction and scattering by a disordered potential. We focus on the low temperature regime, where scattering by phonons is suppressed (see section 2.6. of [24] for a discussion of the temperature range in question). We assume that the correlation length of the disorder potential is large compared to the wavelength of the particles but small compared to the mean free path, and take into account only one feature of the long-range scattering potential to simplify the calculation, the anisotropy in scattering events. The anisotropy, however, is treated exactly. Using this approximation we derive particle and spin-diffusion equations for such systems and investigate the structure of the spin-Hall current.

2. Basic equations

We consider a two-dimensional electron gas with Rashba spin-orbit interaction. The Hamilton operator has the form

$$H = \sum_{\alpha\alpha'} \int \frac{d\mathbf{k}}{(2\pi)^2} (\epsilon_k \delta_{\alpha\alpha'} - (\boldsymbol{\sigma}_{\alpha\alpha'}, \mathbf{N} \times \hbar\mathbf{k})) a_{k\alpha}^\dagger a_{k\alpha'} + \sum_{\alpha} \int \frac{d\mathbf{k}}{(2\pi)^2} \frac{d\mathbf{k}'}{(2\pi)^2} v(\mathbf{k} - \mathbf{k}') a_{k\alpha}^\dagger a_{k'\alpha}. \quad (1)$$

Here $\epsilon_k = \hbar^2 k^2 / (2m)$ is the kinetic energy of the particles in the absence of the spin-orbit interaction, $\mathbf{N} = N e_z$ is a vector perpendicular to the electron gas, (\dots, \dots) symbolizes the scalar product and $a_{k\alpha}^\dagger$ and $a_{k\alpha}$ are creation and annihilation operators for particles with momentum $\hbar\mathbf{k}$ and spin α . The potential v is a disordered potential with zero average and Gaussian statistics. Its standard deviation is given by

$$\langle v(\mathbf{k})v(\mathbf{k}') \rangle_c = (2\pi)^2 \delta(\mathbf{k} + \mathbf{k}') B(\mathbf{k}), \quad (2)$$

where $\langle \dots \rangle_c$ symbolizes the configuration average and $B(\mathbf{k})$ is a slowly varying function of \mathbf{k} . We assume that the disordered potential is of long range in the sense that its correlation length ξ is large compared to the wavelength of the particles but small compared to the mean free path, so the Born approximation is still applicable. The scattering time τ_0 is given by

$$\frac{1}{\tau_0(\epsilon_k)} = \frac{2\pi}{\hbar} \int \frac{d\mathbf{k}_1}{(2\pi)^2} B(\mathbf{k} - \mathbf{k}_1) \delta(\epsilon_k - \epsilon_{k_1}) \quad (3)$$

in this approximation and the momentum relaxation time is given by

$$\frac{1}{\tau_{tr}(\epsilon_k)} = \frac{2\pi}{\hbar} \int \frac{d\mathbf{k}_1}{(2\pi)^2} B(\mathbf{k} - \mathbf{k}_1) \delta(\epsilon_k - \epsilon_{k_1}) (1 - \cos(\phi)), \quad (4)$$

where ϕ is the scattering angle [25, 26]. The description is appropriate for high-mobility heterostructures, in which remote impurity scattering is the main source for the momentum relaxation [26]. For simplicity's sake, we assume that the impurities are separated from the electron gas by a spacer of width z_0 due to δ -doping. The statistics of the impurity potential is nearly Gaussian in such systems [26] and the correlation length is given by the relationship $\xi = z_0/2$.

3. The Green's functions

The propagation of single particle excitations is described by the retarded and advanced Green's functions. They satisfy the Dyson equation

$$[(E - \epsilon_k)\delta_{\alpha\alpha_1} + \hbar(\boldsymbol{\sigma}_{\alpha\alpha_1}, N \times \mathbf{k}) - \Sigma_{\alpha\alpha_1}^{R/A}(\mathbf{k}|E)]G_{\alpha_1\alpha'}^{R/A}(\mathbf{k}|E) = \delta_{\alpha\alpha'}, \quad (5)$$

where $\Sigma^{R/A}$ is the retarded (advanced) self-energy. A summation with respect to double indices has to be performed in equation (5) and also in the equations below. The self-energy is given by

$$\Sigma_{\alpha\alpha'}^R(\mathbf{k}|E) = \int \frac{d\mathbf{q}}{(2\pi)^2} B(\mathbf{k} - \mathbf{q})G_{\alpha\alpha'}^{R/A}(\mathbf{q}|E). \quad (6)$$

In calculating the self-energy we restrict the consideration to its imaginary part. In this case we can take advantage of the fact that the imaginary part of the Green's function is strongly peaked at $E = \epsilon_q$. The width of the peak is of the order of the disorder energy $\hbar/2\tau_0$. The Rashba interaction does not affect this property since we consider only the limit of the weak Rashba interaction, in which the disorder energy is large compared to the Rashba level splitting. The width of the function B is determined by the correlation length ξ . Thus, the peak of the Green's function determines the integral if $\hbar/2\tau_0 \ll \hbar^2/2m\xi^2$. This situation is not uncommon and can always be achieved by choosing the heterostructures properly. Accordingly, we can replace the vector \mathbf{q} in equation (6) by $\mathbf{q} = k_E \mathbf{e}_q$, where $E = \hbar^2 k_E^2/2m$. To simplify the self-energy further we use again the fact that the Green's function is strongly peaked at $E = \epsilon_k$. Using this fact we also replace ϵ_k by E in the imaginary part of the self-energy. The self-energy depends only on e_k and E in this approximation, i.e. $\Sigma^{R/A}(\mathbf{k}|E) \rightarrow \Sigma^{R/A}(e_k|E)$. We note, that this approximation scheme has been used many times before in similar situations (see, e.g., [27] and [28] and references therein).

Our arguments have led us to a self-energy, which depends only on e_k . However, the self-energy is not the only quantity which is simplified. In this approximation the function $B(\mathbf{k} - \mathbf{q})$ is effectively replaced by a function which depends only on E and on the scattering angle, namely $B(\mathbf{k} - \mathbf{q}) \rightarrow B((e_k, e_q)|E)$. The dependence on the scattering angle takes into account the anisotropy in scattering events. Below, we restrict the consideration to systems

with weak anisotropy. In this case we can expand B with respect to the scattering angle and obtain

$$B(\mathbf{k} - \mathbf{q}) \rightarrow B((e_k, e_q)|E) = \frac{\hbar}{2\pi v\tau_0(E)} + \frac{\hbar}{2\pi v\tau_1(E)}(e_k, e_q), \quad (7)$$

where

$$\frac{\hbar}{2\pi v\tau_1(E)} = \frac{1}{\pi} \int_0^{2\pi} d\phi B((e_k, e_q)|E). \quad (8)$$

The quantity τ_1 is connected with the momentum relaxation time τ_{tr} by the relationship

$$\frac{1}{\tau_{tr}} = \frac{1}{\tau_0} - \frac{1}{2\tau_1}. \quad (9)$$

Using equation (7) we eventually obtain

$$\Sigma_{\alpha\alpha'}^R(e_k|E) = -i\frac{\hbar}{2\tau_0(E)}\delta_{\alpha\alpha'} - i\frac{\hbar}{8\tau_1(E)}\sqrt{\frac{2m}{E}}(\boldsymbol{\sigma}_{\alpha\alpha'} \times \mathbf{N}, e_k). \quad (10)$$

The advanced self-energy is obtained from equation (10) by Hermitian conjugation.

The calculations yield a non-trivial self-energy for systems with anisotropic scattering, which differs from that for systems with isotropic scattering. In contrast to systems with isotropic scattering the self-energy depends also on the momentum relaxation time, if anisotropic scattering events occur. At a first glance one is inclined to drop the new term in the self-energy, since it is proportional to the Rashba interaction strength and to $E^{-1/2}$. However, it turns out that this neglect is not justified, since it leads to violation of particle number conservation and to divergences in the solution to the diffusion equations, which occur in the limit of the weak Rashba interaction. Therefore, it is necessary to keep this term. The retarded Green's function has the form

$$G_{\alpha\alpha'}^R(\mathbf{k}|E) = g_+^R(\mathbf{k}|E)g_-^R(\mathbf{k}|E)\left(g_R^{-1}(\mathbf{k}|E)\delta_{\alpha\alpha'} - \hbar\left(\boldsymbol{\sigma}_{\alpha\alpha'} \times \mathbf{N}, \mathbf{k} + i\frac{1}{8\tau_1}\sqrt{\frac{2m}{E}}e_k\right)\right) \quad (11)$$

in this case, where

$$g_{\pm}^R(\mathbf{k}|E) = E - \epsilon_k \pm \Delta^R(\mathbf{k}|E) + i\frac{\hbar}{2\tau_0}, \quad (12)$$

$g_R(\mathbf{k}|E) = g_+^R(\mathbf{k}|E)|_{N=0}$ and

$$\Delta^R(\mathbf{k}|E) = \hbar|N|\left(k + i\frac{1}{8\tau_1}\sqrt{\frac{2m}{E}}\right). \quad (13)$$

4. The diffusion equation

To derive the diffusion equation we use the Keldysh formalism [29]. The magnetization is calculated from the function

$$G_{\alpha\alpha'}^<(\mathbf{k}, \boldsymbol{\kappa}|E, s) = \frac{i}{\hbar} \int_0^{\infty} dt dt' e^{i((E/\hbar+is)t - Et'/\hbar)} \langle a_{\mathbf{k}-\boldsymbol{\kappa}/2\alpha'}^\dagger(t') a_{\mathbf{k}+\boldsymbol{\kappa}/2\alpha'}(t) \rangle \quad (14)$$

in this formalism. Here the bracket $\langle \dots \rangle$ indicates the average with respect to the initial state and s is the Laplace frequency governing the evolution of the particle and spin densities. (To obtain functions at the same instant of time we integrate equation (14) with respect to E and

perform an inverse Laplace transformation with respect to s .) The equation of motion for this quantity can be written in the form [29]

$$\begin{aligned} G_{\alpha\alpha'}^<(\mathbf{k}, \boldsymbol{\kappa}|E, s) - G_{\alpha\alpha_1}^R\left(\mathbf{k} + \frac{\boldsymbol{\kappa}}{2}\middle|E + i\hbar s\right) G_{\alpha_2\alpha'}^A\left(\mathbf{k} - \frac{\boldsymbol{\kappa}}{2}\middle|E\right) \Sigma_{\alpha_1\alpha_2}^<(\mathbf{k}, \boldsymbol{\kappa}|E, s) \\ = G_{\alpha\alpha_1}^R\left(\mathbf{k} + \frac{\boldsymbol{\kappa}}{2}\middle|E + i\hbar s\right) G_{\alpha_2\alpha'}^A\left(\mathbf{k} - \frac{\boldsymbol{\kappa}}{2}\middle|E\right) G_{\alpha_1\alpha_2}^<(\mathbf{k}, \boldsymbol{\kappa}), \end{aligned} \quad (15)$$

where

$$\Sigma_{\alpha\alpha'}^<(\mathbf{k}, \boldsymbol{\kappa}|E, s) = \int \frac{d\mathbf{k}_1}{(2\pi)^2} B(\mathbf{k} - \mathbf{k}_1) G_{\alpha\alpha'}^<(\mathbf{k}_1, \boldsymbol{\kappa}|E, s) \quad (16)$$

is the ‘lesser’ part of the self-energy and $G_{\alpha\alpha'}^<(\mathbf{k}, \boldsymbol{\kappa}) = i\langle a_{\mathbf{k}-\boldsymbol{\kappa}/2, \alpha'}^\dagger(0) a_{\mathbf{k}+\boldsymbol{\kappa}/2, \alpha}(0) \rangle$ is the initial condition.

We are interested in small frequencies ($s\tau_{tr} \ll 1$) and small wavevectors $\boldsymbol{\kappa} = \mathbf{k} - \mathbf{k}'$ in investigating these equations. Therefore, we focus on the limit $s \rightarrow 0$, $\boldsymbol{\kappa} \rightarrow 0$. The right-hand side (rhs) of equation (15) is finite in this limit. There the frequency s competes with the scattering frequency $1/\tau_0$. Consequently, we can take the limit $s \rightarrow 0$ on the rhs and obtain

$$\begin{aligned} G_{\alpha\alpha'}^<(\mathbf{k}, \boldsymbol{\kappa}|E, s) - G_{\alpha\alpha_1}^R\left(\mathbf{k} + \frac{\boldsymbol{\kappa}}{2}\middle|E + i\hbar s\right) G_{\alpha_2\alpha'}^A\left(\mathbf{k} - \frac{\boldsymbol{\kappa}}{2}\middle|E\right) \Sigma_{\alpha_1\alpha_2}^<(\mathbf{k}, \boldsymbol{\kappa}|E, s) \\ = G_{0\alpha\alpha'}^<(\mathbf{k}, \boldsymbol{\kappa}|E). \end{aligned} \quad (17)$$

The quantity

$$G_{0\alpha\alpha'}^<(\mathbf{k}, \boldsymbol{\kappa}|E) = G_{\alpha\alpha_1}^R\left(\mathbf{k} + \frac{\boldsymbol{\kappa}}{2}\middle|E\right) G_{\alpha_2\alpha'}^A\left(\mathbf{k} - \frac{\boldsymbol{\kappa}}{2}\middle|E\right) G_{\alpha_1\alpha_2}^<(\mathbf{k}, \boldsymbol{\kappa}). \quad (18)$$

in this equation is considered as a new initial condition.

Equation (17) yields a convenient starting point for the derivation of diffusion equations. To derive diffusion equations we introduce the distribution functions

$$f_{\alpha\alpha'}(\boldsymbol{\kappa}|E, s) = -i\hbar \int \frac{d\mathbf{k}}{(2\pi)^2} G_{\alpha\alpha'}^<(\mathbf{k}, \boldsymbol{\kappa}|E, s) \quad (19)$$

and

$$f_{\alpha\alpha'}(\boldsymbol{\kappa}|E, s) = -i\hbar \int \frac{d\mathbf{k}}{(2\pi)^2} e_{\mathbf{k}} G_{\alpha\alpha'}^<(\mathbf{k}, \boldsymbol{\kappa}|E, s), \quad (20)$$

where $e_{\mathbf{k}} = \mathbf{k}/k$ is the unit vector in the direction of \mathbf{k} . The particle density $n(\boldsymbol{\kappa}|E, s)$ and the spin density $S(\boldsymbol{\kappa}|E, s)$ are related to these quantities by the relationships

$$n(\boldsymbol{\kappa}|E, s) = f_{\alpha\alpha}(\boldsymbol{\kappa}|E, s) \quad S(\boldsymbol{\kappa}|E, s) = \boldsymbol{\sigma}_{\alpha\alpha'} f_{\alpha'\alpha}(\boldsymbol{\kappa}|E, s). \quad (21)$$

To obtain an equation for the quantity $f_{\alpha\alpha'}$ we integrate equation (17) with respect to \mathbf{k} and use equation (7). This yields the equation

$$\left[\delta_{\alpha\alpha_1} \delta_{\alpha'\alpha_2} - \frac{\hbar}{2\pi\nu\tau_0} I_{\alpha'\alpha_2}^{\alpha\alpha_1}(\boldsymbol{\kappa}|E, s) \right] f_{\alpha_1\alpha_2} - \frac{\hbar}{2\pi\nu\tau_1} (J_{\alpha'\alpha_2}^{\alpha\alpha_1}(\boldsymbol{\kappa}|E, s), f_{\alpha_1\alpha_2}) = \tau_0 f_{0\alpha\alpha'}, \quad (22)$$

where

$$\tau_0 f_{0\alpha\alpha'}(\boldsymbol{\kappa}|E) = -i\hbar \int \frac{d\mathbf{k}}{(2\pi)^2} G_{0\alpha\alpha'}^<(\mathbf{k}, \boldsymbol{\kappa}|E) \quad (23)$$

is the image of the initial condition,

$$I_{\alpha'\alpha_2}^{\alpha\alpha_1}(\boldsymbol{\kappa}|E, s) = \int \frac{d\mathbf{k}}{(2\pi)^2} G_{\alpha\alpha_1}^R\left(\mathbf{k} + \frac{\boldsymbol{\kappa}}{2}\middle|E + i\hbar s\right) G_{\alpha_2\alpha'}^A\left(\mathbf{k} - \frac{\boldsymbol{\kappa}}{2}\middle|E\right) \quad (24)$$

and

$$\mathbf{J}_{\alpha'\alpha_2}^{\alpha\alpha_1}(\boldsymbol{\kappa}|E, s) = \int \frac{d\mathbf{k}}{(2\pi)^2} e_{\mathbf{k}} G_{\alpha\alpha_1}^{\text{R}}\left(\mathbf{k} + \frac{\boldsymbol{\kappa}}{2} \middle| E + i\hbar s\right) G_{\alpha_2\alpha'}^{\text{A}}\left(\mathbf{k} - \frac{\boldsymbol{\kappa}}{2} \middle| E\right). \quad (25)$$

The dependence of the distribution functions on the variables $\boldsymbol{\kappa}$, E and s has been suppressed in equation (22) to avoid cluttering the notation. Since equation (22) is not closed we also need an equation for the function $f_{\alpha\alpha'}$. To obtain such an equation we first multiply equation (17) by $e_{\mathbf{k}}$ and integrate the resulting equation with respect to \mathbf{k} thereafter. Doing so, we obtain

$$\left[\delta_{\alpha\alpha_1} \delta_{\alpha'\alpha_1} \delta_{ij} - \frac{\hbar}{2\pi\nu\tau_1} K_{ij\alpha'\alpha_2}^{\alpha\alpha_1}(\boldsymbol{\kappa}|E, s) \right] f_{j\alpha_1\alpha_2} - \frac{\hbar}{2\pi\nu\tau_0} \mathbf{J}_{\alpha'\alpha_2}^{\alpha\alpha_1} f_{\alpha_1\alpha_2} = \tau_0 f_{0\alpha\alpha'}, \quad (26)$$

where

$$K_{\alpha'\alpha_2}^{\alpha\alpha_1}(\boldsymbol{\kappa}|E, s) = \int \frac{d\mathbf{k}}{(2\pi)^2} e_{\mathbf{k}} \otimes e_{\mathbf{k}} G_{\alpha\alpha_1}^{\text{R}}\left(\mathbf{k} + \frac{\boldsymbol{\kappa}}{2} \middle| E + i\hbar s\right) G_{\alpha_2\alpha'}^{\text{A}}\left(\mathbf{k} - \frac{\boldsymbol{\kappa}}{2} \middle| E\right) \quad (27)$$

and

$$\tau_0 f_{0\alpha\alpha'}(\boldsymbol{\kappa}|E) = -i\hbar \int \frac{d\mathbf{k}}{(2\pi)} e_{\mathbf{k}} G_{0\alpha\alpha'}^<(\mathbf{k}, \boldsymbol{\kappa}|E). \quad (28)$$

is the initial condition. In deriving the diffusion equation we replace the quantity (28) by zero, since knowledge of the initial condition is lost after a time of the order of the transport time.

The equations (22) and (26) yield a closed set of algebraic equations for the calculation of the distribution function $f_{\alpha\alpha'}$. To simplify them we use the fact that we are interested in dirty systems with a weak Rashba spin-orbit interaction. Moreover, we focus on variations of the densities on length scales large compared to the mean free path l ($l\boldsymbol{\kappa} \ll 1$) and restrict the consideration to timescales which are large compared to the momentum relaxation time ($s\tau_{\text{tr}} \ll 1$). In this case we can use the diffusion approximation to simplify our equations [37, 38]. In line with this approximation we expand the quantities $I_{\alpha'\alpha_2}^{\alpha\alpha_1}$, $\mathbf{J}_{\alpha'\alpha_2}^{\alpha\alpha_1}$ and $K_{\alpha'\alpha_2}^{\alpha\alpha_1}$ with respect to $\boldsymbol{\kappa}$ and s . Doing so, we only take into account expansion coefficients, which yield at most contributions of second order in $\boldsymbol{\kappa}$ and of first order in s (see [38] for a discussion of this approximation). Thereafter, we expand the coefficients with respect to N . The latter expansion is motivated by the fact that the Rashba interaction is weak and controlled by the parameter $|\Delta|/(\hbar/2\tau_0) \ll 1$, where $|\Delta|$ is the Rashba level splitting. The calculation of the coefficients on the basis of the quantities $I_{\alpha'\alpha_2}^{\alpha\alpha_1}$, $\mathbf{J}_{\alpha'\alpha_2}^{\alpha\alpha_1}$ and $K_{\alpha'\alpha_2}^{\alpha\alpha_1}$ is elementary but rather lengthy and therefore not presented here. Using this approximation we calculate the function $f_{\alpha\alpha'}$ and insert the result into equation (22). This procedure yields the following diffusion equations after an inverse Laplace transformation with respect to s and a Fourier transformation with respect to $\boldsymbol{\kappa}$

$$\partial_t n - D\Delta n + \Omega\tau_0(\nabla, \mathbf{S} \times \mathbf{N}) = 0, \quad (29)$$

$$\partial_t \mathbf{S} - D\Delta \mathbf{S} - \omega_s(\mathbf{N} \times \nabla) \times \mathbf{S} + \Omega \cdot (\mathbf{S} - \mathbf{S}_0) = 0, \quad (30)$$

$$\mathbf{S}_0 = -\tau_0 \mathbf{N} \times \nabla n. \quad (31)$$

The transport coefficients in these equations are given by

$$D = \frac{E\tau_{\text{tr}}}{m}, \quad \omega_s = 4\frac{m}{\hbar}D, \quad \Omega = 4\frac{N^2m^2}{\hbar^2}D, \quad \Omega_{ik} = \Omega\delta_{ik}(1 + \delta_{i3}). \quad (32)$$

We have ignored a term proportional to $N^2\nabla_x\nabla_y$ in the spin-diffusion equations, although it has a symmetry which differs from that of the terms kept. However, the term ignored is only important in applications far from equilibrium with special boundary conditions, which are outside the scope of our paper.

The equations (29) and (30) have nearly the same form as for systems with short-range scattering [20–22, 30]. The main difference is that the scattering time is replaced by the momentum relaxation time in all transport coefficients except those coupling spin and charge. The spin-accumulation (31) is determined by the scattering time, and not by the momentum relaxation time. The spin-galvanic current, which occurs in the third term on the left-hand side of equation (29), is determined by the coefficient $\Omega\tau_0$ and thus is also related to the scattering time.

We would like to mention that our result for the spin accumulation differs from that of [3]. It would be interesting to compare both calculations. However, it is impossible to do so. The authors of [3] discuss neither the statistical correlations they take into account in their calculation nor the range of their impurity correlation function, the structure of their Green's function or any other step of their calculation. They argue that the characteristic timescale in the spin accumulation is the transport time and refer to the book [31] for the calculation, which, however, is restricted to spinless particles. Our results are also in line with those derived in [31].

The result that the characteristic time in the spin-accumulation is the scattering time is unexpected. Most transport coefficients are determined by the transport time in systems with long-range scattering. One would like to compare the results of the diffusion approximation with an independent calculation in view of this fact. In the appendix we present such a calculation, which has been performed within the framework of the Kubo formula. The results, however, are completely in line with those from the diffusion equation.

5. The spin-Hall current

To calculate the spin-Hall current we extend the method of [21] and [32] to systems with long-range scattering. To this end we focus on systems with concentration gradients and use the expression

$$\hat{J}_{iz} = \frac{1}{2}\{\hat{v}_i, \sigma_z\} \quad (33)$$

for the spin-Hall current operator. Here \hat{v}_i is the i th-component of the velocity operator. Although it is not clear, how the expectation value of the spin-current operator is related to the spin-accumulation in a spin-Hall experiment we expect that a non-vanishing spin-Hall effect in dirty systems only exists in systems with non-vanishing spin-Hall current, since particles running in opposite directions have their spin aligned in opposite z -directions only in such systems. How the spin-Hall current should be measured has recently been discussed in [33, 34].

For the calculation of the spin-Hall current we use the ladder approximation. The expression for the spin-Hall current takes the form

$$\langle \hat{J}_{iz} \rangle = -i\hbar \int \frac{dk_i}{(2\pi)^2} \frac{\hbar k}{m} \sigma_{\alpha'\alpha}^z G_{\alpha\alpha_1}^R \left(\mathbf{k} + \frac{\boldsymbol{\kappa}}{2} \middle| E \right) G_{\alpha_2\alpha'}^A \left(\mathbf{k} - \frac{\boldsymbol{\kappa}}{2} \middle| E \right) \Sigma_{\alpha_1\alpha_2}^<(\mathbf{k}, \boldsymbol{\kappa} | E, s) \quad (34)$$

for large times in this approximation. In investigating this expression we restrict the consideration to the hydrodynamic limit, namely we expand the Green's function with respect to $\boldsymbol{\kappa}$ and retain only contributions which are at most linear in $\boldsymbol{\kappa}$. In calculating the self-energy we use the approximation (7). Due to this fact we can again express the self-energy by the functions $f_{\alpha\alpha'}$ and $\mathbf{f}_{\alpha\alpha'}$ and the quantity $\mathbf{f}_{\alpha\alpha'}$ by the function $f_{\alpha\alpha'}$ by means of equation (26). Using this procedure we obtain after a Fourier transformation

$$\langle J_{iz} \rangle = -D\nabla_i S_z + \frac{1}{2}\omega_s N[(S_x - S_{0x}) + (S_y - S_{0y})]. \quad (35)$$

If we apply a concentration gradient to a homogeneous system, say in the x -direction, we obtain $\nabla_y \rightarrow 0$, $S_x = S_{0x} = 0$ and $S_y = S_{0y}$ in the steady state. Consequently, the spin-Hall current vanishes in the steady state, as expected. The result is in line with the general arguments of the [35] and [36].

6. Conclusions

In this paper we have derived a coupled system of particle and spin-diffusion equations for systems with long-range impurity scattering. We have focused on systems in which the correlation length of the impurity potential is large compared to the wavelength of the particles but small compared to the mean free path. Such systems have the property that they can still be described by means of the conventional Born approximation.

Our investigations have shown that the structure of the diffusion equations is not affected by anisotropy in scattering events in a finite-range impurity potential. The diffusion equations have literally the same structure as for systems with short-range impurity scattering (see, e.g., [20–22, 30]). The scattering time, however, is replaced by the momentum relaxation time in some of the transport coefficients. This replacement affects the diffusion coefficients, the decay rates and the frequency ω_s , which governs the precession of an inhomogeneous magnetization due to the Rashba spin–orbit coupling. Therefore, the spin–spin coupling coefficients and charge–charge coupling coefficients renormalize in the same way as the conductivity. The spin–charge coupling coefficients, however, are affected differently. Our calculation shows that the spin-accumulation due to an electric field is not determined by the momentum relaxation time τ_{tr} but by the scattering time τ_0 . This fact manifests itself also in the spin-galvanic current, which proves to be proportional to $\tau_0\tau_{tr}$.

Most estimates for the magnitude of the spin-accumulation in the literature are based on formulae which have been derived for systems with short-range scattering. The momentum relaxation time agrees with the scattering time in such systems. However, we expect that the spin-accumulation is smaller than previously assumed since the momentum relaxation time is much larger than the scattering time. The difference can be particularly large in high-mobility heterostructures with remote impurity scattering, in which the scatterers are separated from the 2D electron gas by a spacer of width z_0 . The momentum relaxation time is given by

$$\frac{1}{\tau_{tr}} = \nu \int_0^\pi d\theta \int dz |\chi(z)|^2 \frac{\exp(-4k_F|z_0 - z| \sin(\theta/2))}{(2k_F \sin(\theta/2) + q_{sc})^2} (1 - \cos(\theta)) \quad (36)$$

in such systems [39], where ν is a frequency, $\hbar k_F$ is the momentum at the Fermi surface, q_{sc} describes the screening and $\chi(z)$ is the envelope function of the quasi two-dimensional electron gas. The equation for τ_0 differs from equation (36) only in that the factor $(1 - \cos(\theta))$ is absent. This factor is important for high particle densities since the exponent favours small-angle scattering in this case. However, unfortunately it is impossible to obtain a universal numerical estimate for the ratio of the relaxation times since the expression (36) also depends on $\chi(z)$. The integral (36) is very sensitive to the tails of the envelope function, although the tails decay strongly outside the active layer [39]. Therefore, the mobility can be changed considerably by changing their shape. This fact is taken advantage of in velocity-modulated transistors today [39]. Thus, the impact of the tails on the magnitude of the relaxation time is really strong. However, if we take the radical point of view and replace the function $|\chi(z)|^2$ by $\delta(z)$ we find that this ratio is at most of the order of

$$\frac{\tau_{tr}}{\tau_0} \sim (2k_F z_0)^2 = 8\pi n z_0^2, \quad (37)$$

where n is the particle density. Equation (37) yields 56 for $n = 10^{12} \text{ cm}^{-2}$ and $z_0 = 150 \text{ \AA}$. Thus, the fact that the spin-accumulation is determined by the transport time can lead to strong reductions of the magnitude of the spin-accumulation. We would like to mention that a reduction by a factor of 4 has been observed in [5]. Our results might yield an explanation for this.

Finally we would like to draw attention to the fact that anisotropic scattering events also occur in systems in which electron–electron collisions and piezoelectric scattering events are

important. A recent investigation of the impact of electron–electron collisions on the spin relaxation rate Ω indicates that such collisions affect the spin relaxation rate in the same way as collisions with impurities [40]. The results on spin relaxation of [40] agree with those of the present paper. Thus, it might also be possible to observe a reduction of the magnitude of the spin-accumulation in such systems.

Appendix. Spin-accumulation in the Kubo formalism

Here we give an alternative derivation for the spin-accumulation. To calculate the spin-accumulation in a weak electric field we assume that the field \mathcal{E}_x is applied in the x -direction. A non-equilibrium magnetization in the y -direction is obtained in this case [1, 2]. The magnetization is given by the equation [1, 2]

$$S_y = \frac{e\hbar\mathcal{E}_x}{2\pi} \int \frac{d\mathbf{k}}{(2\pi)^2} \left(\frac{\hbar k_x}{m} P_{y0}(\mathbf{k}) - N P_{yy}(\mathbf{k}) \right) \quad (\text{A.1})$$

in the Kubo formulation, where

$$V P_{\alpha\beta}(\mathbf{k}) = \frac{1}{2} \int \frac{d\mathbf{k}'}{(2\pi)^2} \text{tr}(\sigma_\alpha G^R(\mathbf{k}', \mathbf{k}|E) \sigma_\beta G^A(\mathbf{k}, \mathbf{k}'|E))_c, \quad (\text{A.2})$$

V is the volume of the system and e is the charge. The Green's functions in equation (A.2) are those not averaged with respect to disorder. The quantity $P_{\alpha\beta}(\mathbf{k})$ satisfies the Bethe–Salpeter equation

$$P_{\alpha\beta}(\mathbf{k}) = P_{0\alpha\beta}(\mathbf{k}) + \int \frac{d\mathbf{k}_1}{(2\pi)^2} P_{\alpha\gamma}(\mathbf{k}_1) B(\mathbf{k}_1 - \mathbf{k}) P_{0\gamma\beta}(\mathbf{k}), \quad (\text{A.3})$$

where

$$P_{0\alpha\beta}(\mathbf{k}) = \frac{1}{2} \text{tr}(\sigma_\alpha G^R(\mathbf{k}|E) \sigma_\beta G^A(\mathbf{k}|E)). \quad (\text{A.4})$$

To simplify this equation we use again equation (7). Doing so, we find that the solution to equation (A.3) takes the form

$$P_{\alpha\beta}(\mathbf{k}) = P_{0\alpha\beta}(\mathbf{k}) + \frac{\hbar}{2\pi v\tau_0} P_{\alpha\gamma} P_{0\gamma\beta}(\mathbf{k}) + \frac{\hbar}{2\pi v\tau_1} (P_{\alpha\gamma}, \mathbf{e}_k) P_{0\gamma\beta}(\mathbf{k}), \quad (\text{A.5})$$

where

$$P_{\alpha\beta} = \int \frac{d\mathbf{k}}{(2\pi)^2} P_{\alpha\beta}(\mathbf{k}), \quad \mathbf{P}_{\alpha\beta} = \int \frac{d\mathbf{k}}{(2\pi)^2} \mathbf{e}_k P_{\alpha\beta}(\mathbf{k}). \quad (\text{A.6})$$

The quantities $P_{\alpha\beta}$ and $\mathbf{P}_{\alpha\beta}$ satisfy the coupled system of algebraic equations

$$P_{\kappa\lambda} - \frac{\hbar}{2\pi v\tau_0} P_{\kappa\eta} P_{0\eta\lambda} - \frac{\hbar}{2\pi v\tau_1} (P_{\kappa\eta}, \mathbf{P}_{0\eta\lambda}) = P_{0\kappa\lambda}, \quad (\text{A.7})$$

$$\mathbf{P}_{\kappa\lambda} - \frac{\hbar}{2\pi v\tau_0} P_{\kappa\eta} \mathbf{P}_{0\eta\lambda} - \frac{\hbar}{2\pi v\tau_1} (P_{\kappa\eta}, \mathbf{K}_{\eta\lambda}) = \mathbf{P}_{0\kappa\lambda}. \quad (\text{A.8})$$

The quantities $P_{0\kappa\lambda}$ and $\mathbf{P}_{0\kappa\lambda}$ are given by equation (A.6) with $P(\mathbf{k})$ replaced by $P_0(\mathbf{k})$ and

$$\mathbf{K}_{\alpha\beta} = \int \frac{d\mathbf{k}}{(2\pi)^2} \mathbf{e}_k \otimes \mathbf{e}_k P_{0\alpha\beta}(\mathbf{k}). \quad (\text{A.9})$$

To solve this system of equations we consider the limit of weak Rashba interaction and focus on those index combinations, which also enter the spin-accumulation. Doing so, we find

$$P_{yy} = \frac{2\pi v\tau_0}{\hbar} \left[\frac{1}{\Omega\tau_0} - \left(1 - \frac{1}{2} \left(\frac{\tau_0}{\tau_1} \right)^2 \right) \right] \quad (\text{A.10})$$

and

$$P_{y0} = \frac{2\pi v\tau_0}{\hbar} \frac{e_y \times N}{v(E)} \left[\frac{1}{2\Omega\tau_0} + \frac{1}{4} \left(\frac{\tau_0}{\tau_1} \right)^2 \right], \quad (\text{A.11})$$

where $E = mv(E)^2/2$. It would be tempting to simplify the calculations using a relationship of the type

$$\int \frac{d\mathbf{k}}{(2\pi)^2} \frac{\hbar\mathbf{k}}{2m} P_{\alpha\beta}(\mathbf{k}) = v(E) P_{\alpha\beta}.$$

However, it can be shown that such a relationship does not hold. Instead of using this relationship we have to calculate the moments from equation (A.5), which yields

$$\int \frac{d\mathbf{k}}{(2\pi)^2} \mathbf{k} P_{y0}(\mathbf{k}) = \frac{2\pi v\tau_0}{\hbar} \frac{e_y \times N}{\hbar} \left(\frac{m}{\tau_0\Omega} + \frac{m}{2} \left(\frac{\tau_0}{\tau_1} \right)^2 \right). \quad (\text{A.12})$$

Using the equations (A.10) and (A.12) we immediately find

$$S_y = 2ve\tau_0\mathcal{E}N, \quad (\text{A.13})$$

in line with the diffusion equation.

At this point it is important to note the following: we cannot restrict the consideration to the leading approximation in calculating the moments of the correlation functions, since the leading approximation cancels. Thus, we also have to investigate small corrections with respect to the leading approximation. Thus, the method used for the calculation also has to be suitable to adequately take into account higher corrections with respect to the Rashba interaction. The corrections, however, are also determined by the second term on the right-hand side of equation (10). Thus, this contribution is also important in investigations based on the Kubo formalism.

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